UNIT - 1 Functions

Definition of function

A function (f) is a rule correspondence between two sets A & B such that each elements of the set A is assigned to only one elements of the set B

Example

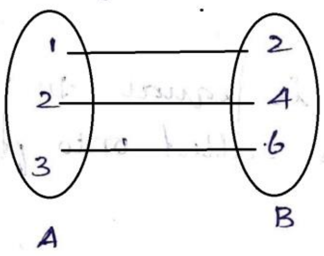
$f(x) = 2x$

$f = 2x$

$f(1) = 2(1) = 2$

$f(2) = 2(2) = 4$

$f(3) = 2(3) = 6$



Odd function

A function $f(x)$ is called an odd function if $f(-x) = -f(x)$

Example : $f(x) = x^3$

$f(-x) = (-x)^3$

$= (-x) \times (-x) \times (-x)$

$= -x^3$

$f(-x) = -f(x)$

Even function

A function $f(x)$ is called an even function. If $f(-x) = f(x)$

Example

$$f(x) = x^4$$

$$f(-x) = (-x)^4$$

$$= (-x) \times (-x) \times (-x) \times (-x)$$

1) Let f be a function defined by $f(x) = x^2 + 1$ find the range of f

Sol Given $v = [-2, -1, 0, 1, 2]$

$f(x) = x^2 + 1$

$f(-2) = (-2)^2 + 1$

$= 4 + 1$

$\boxed{f(-2) = 5}$

$f(-1) = (-1)^2 + 1$

$= 1 + 1$

$\boxed{f(-1) = 2}$

$f(0) = (0)^2 + 1$

$= 0 + 1$

$= 1$

$f(1) = 1^2 + 1$

$= 1 + 1$

$\boxed{f(1) = 2}$

$f(2) = 2^2 + 1$

$= 4 + 1$

$\boxed{f(2) = 5}$

$f = x^2 + 1$

The range $f = [1, 2, 5]$

2) Let $v = [-3, -1, 0, 1, 3]$

$f(x) = x^2 - 1$

Let f function defined by $f(x) = x^2 - 1$ find the range of f

Sol Given

$v = [-3, -1, 0, 1, 3]$

$f(x) = x^2 - 1$

$f(-3) = (-3)^2 - 1$

$= 9 - 1$

$\boxed{f(-3) = 8}$

$f(-1) = (-1)^2 - 1$

$= 1 - 1$

$\boxed{f(-1) = 0}$

$f(0) = (0)^2 - 1$

$\boxed{f(0) = -1}$

$f(1) = (1)^2 - 1$

$= 1 - 1$

$\boxed{f(1) = 0}$

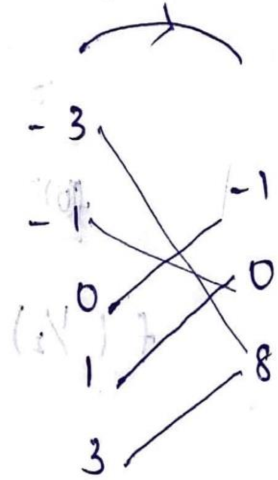
$f(3) = (3)^2 - 1$

$= 9 - 1$

$= 8$

The range of $f = [-1, 0, 8]$

$f = x^2 - 1$



3) If $f(x) = 3x^2 - 2x + 5$

find $f(-2)$ $f(2)$ $f(0)$ $f(1/2)$

Sol Given

$$f(x) = 3x^2 - 2x + 5$$

$$f(-2) = 3(-2)^2 - 2(-2) + 5$$

$$= 3(4) + 4 + 5$$

$$= 12 + 9$$

$$\boxed{f(-2) = 21}$$

$$f(2) = 3(2)^2 - 2(2) + 5$$

$$= 3(4) - 4 + 5$$

$$= 12 - 1$$

$$\boxed{f(2) = 13}$$

$$f(0) = 3(0)^2 - 2(0) + 5$$

$$= 0 - 0 + 5$$

$$\boxed{f(0) = 5}$$

$$f(0) = 3(0)^2 - 2(0) + 5$$
$$= 0 - 0 + 5$$
$$\boxed{f(0) = 5}$$

$$f(1/2) = 3(1/2)^2 - 2(1/2) + 5$$
$$= 3(1/4) - 1 + 5$$
$$= 3/4 + 4 = \frac{3 + 16}{4}$$
$$\boxed{f\tfrac{1}{2} = 19/4}$$

Ans: $f(-2) = 21$

$f(2) = 13$

$f(0) = 5$

$f(1/2) = 19/4$

4) $f(x) = 3x^2 - 2x + 5$

find $= f(-2),\ f(2)\ f(1)\ f(-1)$

Sol

Given

$f(x) = 3x^2 - 2x + 5$

$f(-2) = 3(-2)^2 - 2(-2) + 5$

$= 3(4) + 4 + 5$

$= 12 + 9$

$\boxed{f(-2) = 21}$

$f(2) = 3(2)^2 - 2(2) + 5$

$= 3(4) - 4 + 5$

$= 12 + 1$

$\boxed{f(2) = 13}$

$f(1) = 3(1)^2 - 2(1) + 5$

$= 3 - 2 + 5$

$\boxed{f(1) = 6}$

$$f(-1) = 3(-1)^2 - 2(-1) + 5$$
$$= 3 + 2 + 5$$
$$\boxed{f(-1) = 10}$$

Ans: $f(-2) = 21$

$f(2) = 13$

$f(1) = 6$

$f(-1) = 10$

5) $f(x) = 5x^2 - 2x + 3$

find $f(-2)$ $f(2)$ $f(1)$ $f(3)$

Sol Given

$f(x) = 5x^2 - 2x + 3$

$f(-2) = 5(-2)^2 - 2(-2) + 3$

$= 5(4) + 4 + 3$

$= 20 + 7$

$\boxed{f(-2) = 27}$

$f(2) = 5(2)^2 - 2(2) + 3$

$= 5(4) - 4 + 3$

$= 20 - 4 + 3$

$\boxed{f(2) = 19}$

$$f(1) = 5(1)^2 - 2(1) + 3$$
$$= 5 - 1 + 3$$
$$\boxed{f(1) = 7}$$

$$f(3) = 5(3)^2 - 2(3) + 3$$
$$= 5(9) - 6 + 3$$
$$= 45 - 6 + 3$$
$$f(3) = 42$$

Ans : $f(-2) = 27$

$f(2) = 19$

$f(1) = 7$

$f(3) = 42$

6) If $f(x) = 2x^2 - 5x + 4$ for what value of x $2f(x) = f(2x)$

Solution

Given $f(x) = 2x^2 - 5x + 4$

To find $2f(x)$

$$2f(x) = 2(2x^2 - 5x + 4)$$

$$\boxed{2f(x) = 4x^2 - 10x + 8}$$

To find $f(2x)$

$$f(x) = 2x^2 - 5(2x + 4$$

$$f(2x) = 2(2x)^2 - 5(2x) + 4$$

$$= 2(4x^2) - 10x + 4$$

$$\boxed{f(2x) = 8x^2 - 10x + 4}$$

$$2f(x) = f(2x)$$

$$4x^2 - 10x + 8 = 8x^2 - 10x + 4$$

$$4x^2 - 10x + 8 - 8x^2 + 10x - 4 = 0$$

$$-4x^2 + 4 = 0$$

$$-4x^2 = -4$$

$$x^2 = \frac{-4}{-4}$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

$\therefore$ If $x = 1$ the given function $f(x)$ $2x^2 - 5x + 4$ satisfy the condition $2f(x) = f(2x)$

7) If $f(x) = (x-1)(x-2)(x-3)$ find the values of $f(1)$ $f(4)$ $f(-1)$

Solution

Given $f(x) = (x-1)(x-2)(x-3)$

to find $f(1)$

$f(1) = (1-1)(1-2)(1-3)$

$= (0)(-1)(-2)$

$\boxed{f(1) = 0}$

to find $f(4)$

$f(4) = (4-1)(4-2)(4-3)$

$= (3)(2)(1)$

$\boxed{f(4) = 6}$

to find $f(-1)$

$f(-1-1)(-1-2)(-1-3)$

$= (-2)(-3)(-4)$

$\boxed{f(-1) = -24}$

$$\boxed{\begin{aligned} f(1) &= 0 \\ f(4) &= 6 \\ f(-1) &= -24 \end{aligned}}$$

8) If $f(x) = (x-1)(x-2)(x-3)$ find the values of $f(3)$ $f(4)$ $f(-1)$

Solution

Given $f(x) = (x-1)(x-2)(x-3)$

To find $f(3)$

$f(3) = (3-1)(3-2)(3-3)$

$= (2)(1)(0)$

$\boxed{f(3) = 0}$

To find $f(4)$

$f(4) = (4-1)(4-2)(4-3)$

$= (3)(2)(1)$

$\boxed{f(4) = 6}$

To find $f(-1)$

$f(-1) = (-1-1)(-1-2)(-1-3)$

$= (-2)(-3)(-4)$

$\boxed{f(-1) = -24}$

$f(3) = 0$

$f(4) = 6$

$f(-1) = -24$

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Q) If $f(x) = (x+1)(x+2)(x+3)$ find the values of $f(1)$ $f(2)$ $f(-3)$

Solution

Given $f(x) = (x+1)(x+2)(x+3)$

To find $f(1)$

$f(1) = (1+1)(1+2)(1+3)$

$= (2)(3)(4)$

$\boxed{f(1) = 24}$

$f(2) = (2+1)(2+2)(2+3)$

$= (3)(4)(5)$

$\boxed{f(2) = 60}$

$f(-3) = (-3+1)(-3+2)(-3+3)$

$= (-2)(-1)(0)$

$\boxed{f(-3) = 0}$

Ans :

$f(1) = 24$

$f(2) = 60$

$f(-3) = 0$

Linear functional

This is the equation of the straight line passing

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

Example 1

The salary of the employee in 1985 was ₹ 1200 in 1987 it will be a ₹ 1350 express salary as linear function of time and estimate his salary in 1988

Solution

Given Let x represents year and y represents salary

1985 (x_1) 1200 (y_1)

1987 (x_2) 1350 (y_2)

1988 (x) ? (y)

By the linear equation

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{y - 1200}{1988 - 1985} = \frac{1200 - 1350}{1985 - 1987}$$

$$\frac{y - 1200}{3} = \frac{+150}{+2}$$

$$\frac{y-1200}{3} = 75$$

$$y - 1200 = 3 \times 75$$

$$y - 1200 = 225$$

$$y = 225 + 1200$$

$$\boxed{y = 1425 \text{ Rs}}$$

salary in 1988 is $\boxed{y = 1425 \text{ Rs}}$

Example 2

The salary of an employee in 1993 was ₹ 1100 in 1995 it will be ₹ 1250 express salary as linear function of prime and estimate his salary in 1998

Sol Given Let $x \to$ represents year
$y \to$ represents salary

1993 (x_1) 1100 (y_1)
1995 (x_2) 1250 (y_2)
1998 (x) ? (y)

By the linear equation

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{y - 1100}{1998 - 1993} = \frac{1100 - 1250}{1993 - 1995}$$

$$\frac{y - 1100}{5} = \frac{-150}{-2}$$

$$\frac{y - 1100}{5} = 75$$

$$y - 1100 = 75 \times 5$$

$$= 375$$

$$y = 375 + 1100$$

$$\boxed{y = 1475}$$

His salary in 1998 is 1475

Example 3

The demand curve for the commodity is $x = 10 - \frac{y}{4}$ where y is the price per unit and x is the number of unit demand

i) find the quantity demand if the price is ₹ 4

ii) find the price if the quantity demand ₹ 7

iii) What quantity we demanded if the commodity were free

Sol Given $x = 10 - \frac{y}{4}$

Let → x represents quantity

→ y represents price

i) If price $y = 4$ x?

$x = 10 - \frac{\cancel{4}}{\cancel{4}}$

$x = 10 - 1$

$\boxed{x = 9 \text{ units}}$

If price is ₹ 4 the quantity demand is 9 units

ii) find price y ? if $x = 7$

$$7 = 10 - \frac{y}{4}$$

$$7 - 10 = -y/4$$

$$+3 = \frac{y}{4}$$

$$3 \times 4 = y$$

$$\boxed{y = 12 \text{ Rs.}}$$

If quantity demand is 7 the price is ₹ 12

iii) If $y = 0$

$$x = 10 - \frac{0}{4}$$

$$\boxed{x = 10 \text{ units}}$$

If commodity when free the no of units is 10

Example 4

The life expentancy of males in 1987 in a country is 70 years in 1952 it was 60 years assuming the life expentance to be a linear function of time make a prediction of life expentancy of males in that country in 1997.

Sol Given

Let $x \rightarrow$ represents year

$y \rightarrow$ represents age

1962 (x_1) 60 (y_1)

1989 (x_2) 70 (y_2)

1997 (x) ? (y)

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{y - 60}{1997 - 1962} = \frac{60 - 70}{1962 - 1987}$$

$$\frac{y - 60}{35} = \frac{-\cancel{10}^{2}}{-\cancel{25}^{5}}$$

$$\frac{y - 60}{35} = \frac{2}{5}$$

$$y - 60 = \frac{2 \times \cancel{35}^{7}}{\cancel{5}}$$

$$y - 60 = 14$$

$$y = 14 + 60$$

$$\boxed{y = 74 \text{ years}}$$

Example 5

The life expendancy of males in 1987 in a country is 75 years in 1962 it was 65 year assuming the life expendancy to be year linear function of the time make a prediction of life expendancy of males in that country in 1997

Sol Given $x \rightarrow$ represents year

$y \rightarrow$ represents age

1962 (x_2)	65 (y_2)
1987 (x_1)	75 (y_1)
1997 (x)	? (y)

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\frac{y-75}{1997-1987} = \frac{75-65}{1997-1962}$$

$$\frac{y-75}{10} = \frac{\cancel{10}\,2}{\cancel{35}\,5}$$

$$y-65 = \frac{2\times \cancel{35}\,7}{\cancel{5}}$$

$$y-65 = 14$$

$$y = 14+65$$

$$\boxed{y = 79 \text{ years}}$$

The life expectation of made in 1997 is 79 years

Quadratic function

A function of a form $f(x) = ax^2 + bx + c$ where a, b and c are constants

Find maximum or minimum points on the graph of a function we proceed as fallows

Step 1 Find the values of A, B and C

Step 2 Determine rather $a > 0$ (or) $a < 0$

Step 3 If $A > 0$ the minimum point occurs at $x = \frac{-b}{2a}$, subtituing this value of x is $f(x)$ we get the minimum value of $f(x)$

$a < 0$ the maximum point occurs at $x = \frac{-b}{2a}$, substuting this value of ~~f(x)~~ x in $f(x)$ we get the maximum value of $f(x)$.

Find the manufacture the cost per unit of manufacturing a certain commodity $y = x^2 - 20x + 200$, when 5 to 200 units are produced per day find the number of units to be produced when the cost is least and also find the cost per unit

Sol Given $y = x^2 - 20x + 200$

compare to this equation to $ax^2 + bx + c$

$a = 1, \quad b = 20, \quad c = 200.$

Here $a > 0$ ∴ the cost per unit is minimum

$$x = \frac{-b}{2a}$$

$$x = \frac{-(20)}{2 \times 1}$$

$$x = \frac{20}{2}$$

$$\boxed{x = 10 \text{ units}}$$

∴ for minimum cost the manufacture should produce atleast 10 units

To find cost per unit

substitude $x = 10$

$$y = x^2 - 20x + 200$$

$$= 10^2 - 20(10) + 200$$

$$= 100 - 2\cancel{0}0 + \cancel{2}00$$

$$\boxed{y = 100 \text{ Rs}}$$

The minimum cost per unint is 100

Find the manufacture the cost per unit of manufacture a certain commodity $y = x^2 - 25x + 250$, when 5 to 200 units are produced per day find the number of units to be produced when the cost is least and also find the cost per unit sol Given $y = x^2 - 25x + 250$

compare to this equation to $ax^2 + bx + c$

$a = 1, \quad b = 25, \quad c = 200$

Here $a > 0$ $\therefore$ the cost per unit is minimum

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-25)}{2(1)} \qquad x = \frac{25}{2}$$

$x = 12.5$, $x = 13$ units

$\therefore$ for minimum cost the manufacture should produce atleast 13 units

To find cost per units

Substitude $x = 13$ in the given equatio

$y = x^2 - 25x + 250$

$y = 13^2 - 25(13) + 250$

$y = 169 - 325 + 250$

$y = 94$ Rs

$\therefore$ The minimum cost per unit is 94 Rs.

A TV manufacture determines that this total cost for producing x TV sets = Is given by $c(x) = 500x + 3000 + 4000$

i) Each set q sales for £ 6000.

ii) Determine the break even point

iii) The nomq sets to be produced to no loss

iv) The nomq sets to be produced to no profit

Sol Given cost function

$C(x) = 500x^2 + 3000x + 4000$

Revenue function

$R(x) = 6000x$

For break even points

$R(x) = C(x)$

$500x^2 + 3000x + 4000 = 6000x$

$500x^2 + 3000x + 4000 - 6000x = 0$

$500x^2 + 3000x + 4000 = 0$

$\div 500 \quad x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2$ or $x = 4$

For the break even point the manufacture should produce 2 set or 4 TV sets

profit function

$P(x) = R(x) - C(x)$

$= 6000x - (500x^2 + 3000x + 4000)$

$= 6000x - 500x^2 - 3000x - 4000$

$= 500x^2 + 3000x - 4000$

$\div 500$

$$-x^2 + 6x - 8 = 0$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

For no loss

$$P(x) \geq 0$$

$$-500x^2 + 3000x - 4000 \geq 0$$

$\div 500$

$$-x^2 + 6x - 8 \geq 0$$

$$x^2 - 6x + 8 \geq 0$$

$(x \text{ by } -1)$

$$-x^2 + 6x - 8 \geq 0$$

$$x^2 - 6x + 8 \geq 0$$

$(x \text{ by } -1)$

$$x^2 - 6x + 8 \leq 0$$

$$(x-2)(x-4) \leq 0$$

$$2 \leq x \leq 4$$

For no loss the manufacture should produce 2 or more set or 4 or less TV sets

For a manufacture of dry cells a daily cost of production 'c' for cells is given by $c(x) = 2.05x + 750$ If the prize of a cell is ₹ 3 Determine the minimum no. of cells that must be produced and sold to no loss.

Sol Given

The cost function $c(x) = 2.50x + 750$

Revenue function

$R(x) = 3x$

profit function

$P(x) = R(x) - c(x)$

$P(x) = 3x - (2.05x + 750)$

$= 3x - 2.05x - 750$

$= 0.95x - 750$

For no loss

$P(x) \geq 0$

$0.95x \geq 0$

$0.95x \geq 750$

$x \geq \frac{750}{0.95}$

$x \geq 789.47$

$x \geq 790$

For no loss the company or sell at least 700 prices

For a daily manufacture of dry cells a daily cost of production 'c' for cells is given by $c(x) = 0.25x + 550$. If the price of a cell is ₹ 3. determine the no. of cells that must be produced and sold to no loss

Sol Given

cost function

$c(x) = 2.05x + 550$

Revenue function

$R(x) = 3x$

Profit function

$p(x) = R(x) - c(x)$

$3x - (2.05x + 550)$

$3x - 2.05x - 550$

$p(x) = 0.95x - 550$

For no loss

$p(x) \geq 0$

$0.95x - 550 \geq 0$

$0.95x \geq 550$

$x \geq \frac{550}{0.95}$

$x \geq 578.94 \quad x \geq 580$

The daily cost of production 'c' for x units of an assembling is given by $c(x) = 12.5x + 6400$

1. If each units is sold for ₹ 25 determine the minimum no. of units that should be produced and sold that ensure no loss

2. If the selling price is reduced by ₹ 2.5 per units what could be the break even point

Sol Given

Let x represents camara

y represents price

500 (y_1) 50 (x_1)

750 (y_2) 100 (x_2)

? (y) 150 (x)

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\frac{y-500}{150-50} = \frac{500-750}{50-100}$$

$$\frac{y-500}{100} = \frac{\cancel{250}^{5}}{\cancel{50}}$$

$$\frac{y-500}{100} = 5$$

$$y-500 = 5 \times 100$$

$y = 500 + 500$

$y = 1000$ Rs

If 150 camaras are made available then expect price per camaras is ₹ 1000

When the price is ₹ 500 60 mobiles are fixed type were available for sale when price was 720 mobiles were available what is the supply equation assuming that it is linear if 180 mobiles were made available what is the escpected price for mobile

Sol

Given Let x represents mobile

y represents price

$500\ (y_1)$ $60\ (x_1)$

$750\ (y_2)$ $120\ (x_2)$

$?\ (y)$ $180\ (x)$

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{y_1 - 500}{180 - 60} = \frac{500 - 750}{60 - 120}$$

$$\frac{y - 500}{120} = \frac{-250}{-60}$$

$$y - 500 = \frac{-250}{-60} 120$$

$$y - 500 = 500$$

$$y = 500 + 500$$

$$y = 1000$$

If the 180 mobiles are made available then the expected price per mobiles is 1000

When the price is ₹ 750, 50 computers are fixed type were available for sale when price was 1000, 100 computers were available what is the supply equation assuming that it is linear if 150 computers are made available what is the expected price per computer?

Sol Given

Let x represents computers

y represents amount

$750\ (y_1)$ $50\ (x_1)$

$1000\ (y_2)$ $100\ (x_2)$

$?\ (y)$ $150\ (x)$

By using linear equation

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\frac{y-750}{150-50} = \frac{750-1000}{50-100}$$

$$\frac{y-750}{100} = \frac{+\cancel{250}^{5}}{+\cancel{50}}$$

$$y - 750 = 5 \times 100$$

$$y - 750 = 500$$

$$y = 500 + 750$$

$$\boxed{y = 1250}$$

$\therefore$ If 150 computers are available, the price per computer 1250 ₹